



An introduction to energy-economic computable general equilibrium model

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Supplementary notes:
An illustrative energy-economic computable general equilibrium model

Overview

1. A simple energy-economic model (ENERGY1 .gms)
2. An energy model with emissions and a cap-and-trade policy (ENERGY2 .gms)

A simple energy-economic model (ENERGY1.gms)

- ENERGY1.gms identifies five energy sectors (coal, crude oil, refined oil, gas and electricity) and groups other sectors in a single sector (CO₂ emissions are not considered at this stage)

```
SET i Sectors /  
    cru      Crude oil  
    oil      Refined oil  
    col      Coal  
    gas      Gas  
    ele      Electricity  
    oth      Other output/;  
  
SET f Factors of production /  
    lab      Labor  
    cap      Capital  
    res      Natural resources /;
```

```
ALIAS (i,j);
```

```
SET ele(i) Electricity      /ele/;  
SET ff(i)   Fossil fuels    /cru,col,gas/;  
SET fe(i)   Final energy    /oil,col,gas/;  
SET en(i)   Energy markets  /gas,ele,oil,col,cru/;
```

Subsets to assist model specification

A simple energy-economic model (ENERGY1.gms)

- Data for the model is sourced from edata.gms

```
$include edata.gms
```

Input/output	Sector						
	Gas	Electricity	Oil	Coal	Crude Oil	OTHR	HH
Gas	150	-60				-70	-20
Electricity		850				-750	-100
Oil		-30	500			-460	-10
Coal		-35		50		-10	-5
Crude Oil			-300		300		
OTHR	-65	-325	-175	-15	-170	24790	-24040
Capital	-50	-300	-15	-10	-70	-9500	9945
Labor	-20	-100	-10	-10	-10	-14000	14150
Gas resource	-15						15
Coal resource				-15			15
Crude oil resource					-50		50

A simple energy-economic model (ENERGY1.gms)

- Variable assignments in ENERGY1

```
$SECTORS:
    W                                ! Welfare index
    Y(i) $vom(i)                    ! Output index

$COMMODITIES:
    PW                              ! Welfare price index
    PY(i) $vom(i)                  ! Output price index
    PL                              ! Labor price index
    PK                              ! Capital price index
    PR(i) $vfm("res",i)           ! Natural resources price index

$CONSUMERS:
    HH                              ! Representative household
```

- Production in each sectors is a multi-level next of constant elasticity of substitution functions

A simple energy-economic model (ENERGY1.gms)

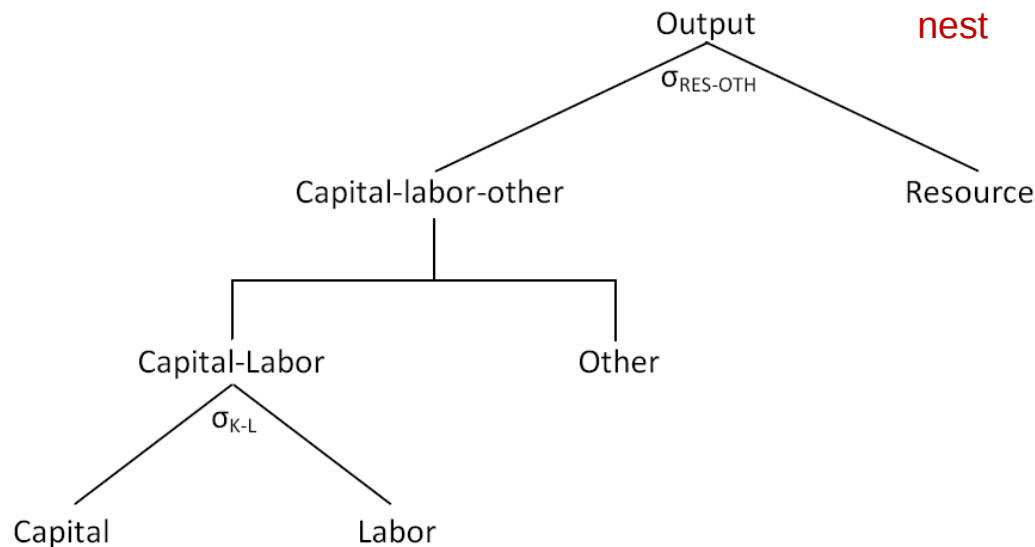
The 'b' nest is a sub-nest of 's' by default

'va' is assigned as a sub-nest of 'b'

```
*      Fossil fuel production (crude oil, coal, gas)
$PROD:Y(ff)$vom(ff) s:sigma("res_oth") b:0 va(b):sigma("k_1")
O:PY(ff)      Q:vom(ff)
I:PY(j)      Q:vfi(j,ff)
I:PL      Q:vfm("lab",ff)
I:PK      Q:vfm("cap",ff)
I:PR(ff)      Q:vfm("res",ff)
```

```
b: a:HH T:tf(j,ff)
va:
va:
```

Nesting assignments – unless otherwise noted, each input is assigned to the top level (s) nest



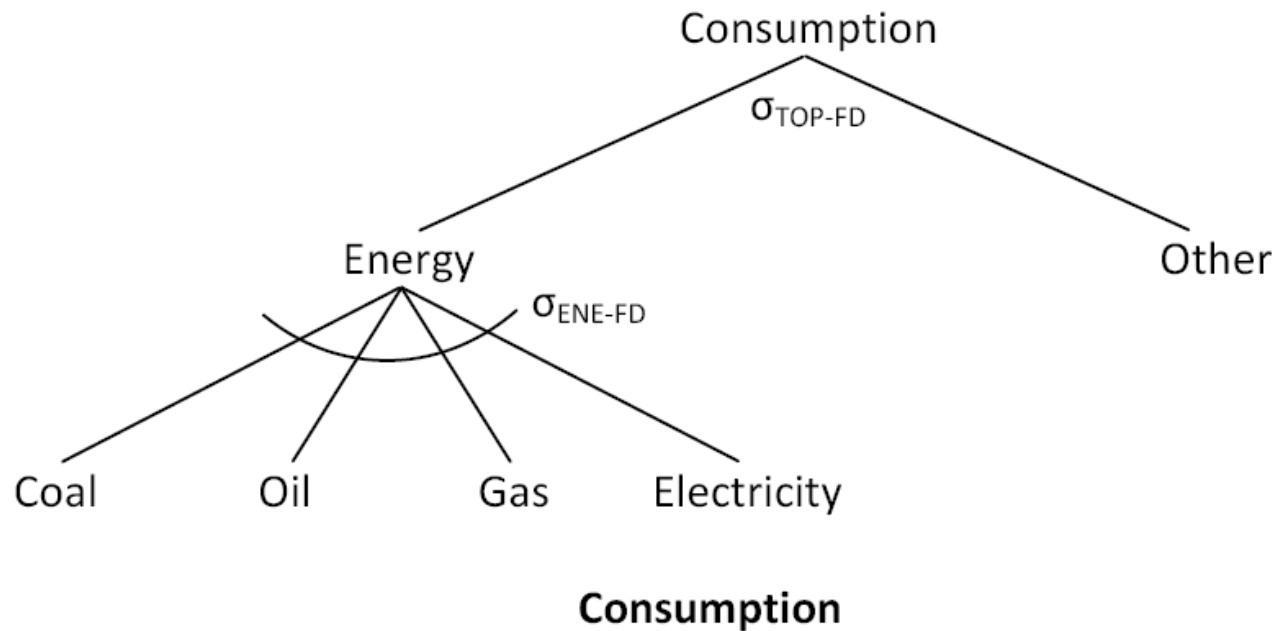
Fossil fuels

A simple energy-economic model (ENERGY1.gms)

```
*      Consumer welfare function
$PROD:W s:sigma("top_fd") e:sigma("ene_fd")
      O:PW                      Q:vthp
      I:PY(i)$ (not fe(i))      Q:vhp(i)          e:$ele(i)
      I:PY(fe)                  Q:vhp(fe)          e:
```

A simple energy-economic model (ENERGY1.gms)

```
*      Consumer welfare function
$PROD:W s:sigma("top_fd") e:sigma("ene_fd")
O:PW                                Q:vthp
I:PY(i)$(not fe(i))                Q:vhp(i)          e:$ele(i)
I:PY(fe)                            Q:vhp(fe)         e:
```



A simple energy-economic model (ENERGY1.gms)

* Electricity production

```
$PROD:Y(i)$ (vom(i) and ele(i)) s:0 kle:sigma("e_kl") va(kle):sigma("k_l")
+ ene(kle):sigma("fe")
```

O:PY(i)

Q:vom(i)

I:PY(j)\$ (not fe(j))

Q:vfi(j,i)

a:HH T:tf(j,i)

I:PL

Q:vfm("lab",i)

va:

I:PK

Q:vfm("cap",i)

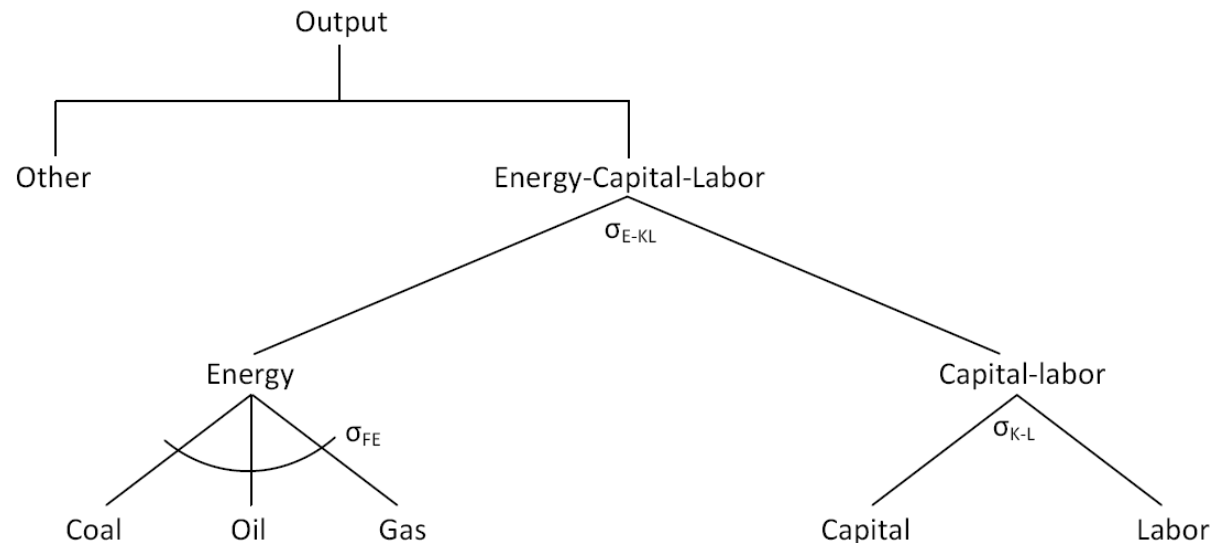
va:

I:PY(fe)

Q:vfi(fe,i)

ene:

a:HH T:tf(fe,i)

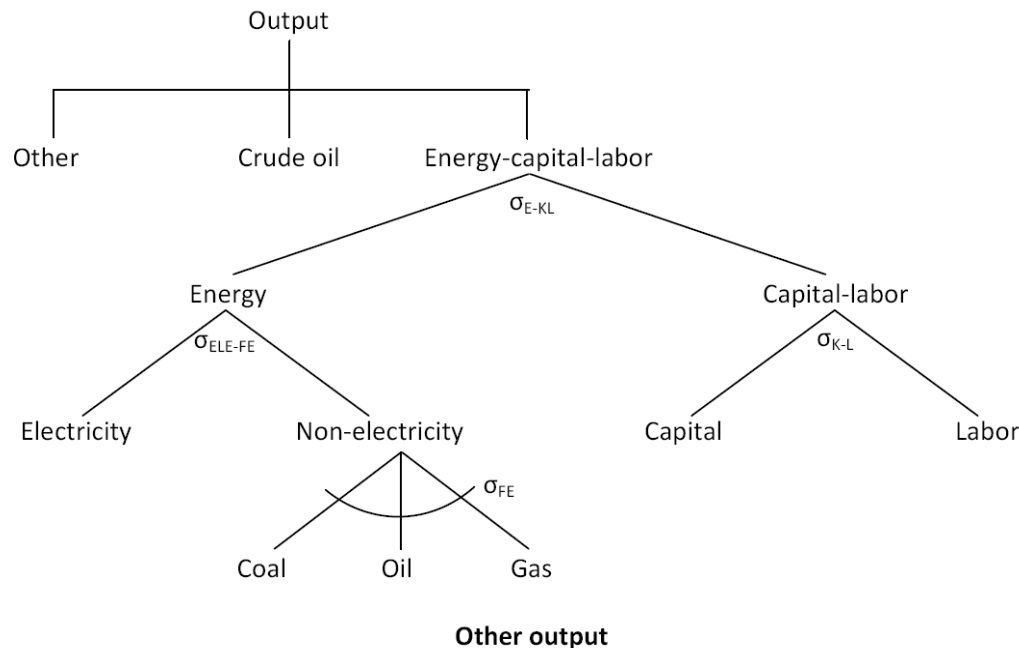


Electricity

A simple energy-economic model (ENERGY1.gms)

```

*      Other output and refined oil production
$PROD:Y(i)$ (vom(i) and not ele(i) and not ff(i)) s:0 kle(s):sigma("e_kl")
+ va(kle):sigma("k_l") e(kle):sigma("ele_fe") ne(e):sigma("fe")
      O:PY(i)                                Q:vom(i)
      I:PY(j)$ (not fe(j))                  Q:vfi(j,i)      e:$ele(j) a:HH T:tf(j,i)
      I:PL                                  Q:vfm("lab",i)    va:
      I:PK                                  Q:vfm("cap",i)    va:
      I:PY(fe)                             Q:vfi(fe,i)      ne:      a:HH T:tf(fe,i)
  
```



A simple energy-economic model (ENERGY1.gms)

- ENERGY1 implements a 50% tax on coal used as an intermediate input and reports changes in output and welfare.

```
tf("col",i) = 0.5;
```

```
$include ENERGY1.gen  
solve ENERGY1 using mcp;
```

```
PARAMETER  
OUTPUT(*,i)           Output  
WELFARE(*)             Welfare;
```

```
OUTPUT("BASE",i) = vom(i);  
OUTPUT("SIM",i) = vom(i)*Y.L(i);  
OUTPUT("%C",i) = 100*(OUTPUT("SIM",i) - vom(i))/vom(i);
```

```
WELFARE("%C") = 100*(W.L - 1);
```

```
DISPLAY OUTPUT, WELFARE;
```

A simple energy-economic model (ENERGY1.gms)

- When operating large-scale models, sending output to a.gdx file allows the user to view variable and parameter values and definitions (if included) easily. ENERGY1 sends all model output to ENERGY1_OUTPUT.gdx.

```
EXECUTE_UNLOAD "ENERGY1_OUTPUT.gdx"
```

Exercises

- Implement a 50% tax on coal used for electricity production without taxes on coal elsewhere.
- When there is a 50% tax on coal used in all sectors, change the value of `sigma("fe")` from 1.2 to 2 and interpret the results.

An energy model with emissions (ENERGY2.gms)

- ENERGY2 adds CO₂ emissions from fossil fuels to ENERGY1 and considers a cap-and-trade policy
- The level of the cap is represented by the parameter `clim` and an emissions price declaration is included under the `$COMMODITIES` command

PARAMETER

<code>fico2(j,i)</code>	CO2 emission by fuel for each sector - million metric tons
<code>hhco2(i)</code>	CO2 emissions from household consumption by fuel type - million metric tons
<code>clim</code>	CO2 emissions limit;

`clim = 0;`

`$COMMODITIES:`

`PCO2$clim`

Setting `clim = 0` turns off the carbon constraint (to replicate the benchmark equilibrium)

! Price of co2 emissions

An energy model with emissions (ENERGY2.gms)

- Emissions are introduced by applying fixed carbon coefficients per unit of energy from oil, coal and gas
- Producers and households are required to purchase an emissions allowance for each ton of CO₂ emissions

Oil and CO₂ allowances are included in a 'Leontief' (fixed proportions) nest

```
*      Consumer welfare function
$PROD:W s:sigma("top_fd") e:sigma("ene_fd") oil(e):0 col(e):0 gas(e):0
      O:PW                                Q:vthp
      I:PY(i)$(not fe(i))                Q:vhp(i)                                e:$ele(i)
      I:PY("oil")                        Q:vhp("oil")                                oil:
      I:PCO2$clim                        Q:hhco2("oil")        P:1e-5      oil:
      I:PY("col")                        Q:vhp("col")                                col:
      I:PCO2$clim                        Q:hhco2("col")        P:1e-5      col:
      I:PY("gas")                        Q:vhp("gas")                                gas:
      I:PCO2$clim                        Q:hhco2("gas")        P:1e-5      gas:
```

No emissions price in the base data and a CES function calibrated to a commodity with a zero cost share will always be zero, so the "P:" field is used to assign a "small" positive price

An energy model with emissions (ENERGY2.gms)

- A more succinct but equivalent representation of the consumer welfare function

```
*      Consumer welfare function
$PROD:W s:sigma("top_fd") e:sigma("ene_fd") oil(e):0 col(e):0 gas(e):0
O:PW                                Q:vthp
I:PY(i)$(not fe(i))                Q:vhp(i)                e:$ele(i)
I:PY(fe)                            Q:vhp(fe)                fe.tl:
I:PCO2#(fe)$clim Q:hhco2(fe)        P:1e-5                fe.tl:
```

Instructs GAMS to include a separate `I:PCO2#(fe)` for each element of `fe`

The `".tl"` suffix instructs GAMS to display each element of `fe`

An energy model with emissions (ENERGY2.gms)

- Similar expression are used to include emissions in other production functions

```
*      Electricity production
$PROD:Y(i)$ (vom(i) and ele(i)) s:0 kle:sigma("e_kl") va(kle):sigma("k_l")
+ ene(kle):sigma("fe") oil(ene):0 col(ene):0 gas(ene):0
      O:PY(i)                      Q:vom(i)
      I:PY(j)$ (not fe(j))         Q:vfi(j,i)
      I:PL                          Q:vfm("lab",i)      va:
      I:PK                          Q:vfm("cap",i)      va:
      I:PY(fe)                     Q:vfi(fe,i)          fe.tl:
      I:PCO2#(fe)$clim             Q:fico2(fe,i)      P:1e-5      fe.tl:
```

```
*      Other output and refined oil production
$PROD:Y(i)$ (vom(i) and not ele(i) and not ff(i)) s:0 kle(s):sigma("e_kl")
+ va(kle):sigma("k_l") e(kle):sigma("ele_fe") ne(e):sigma("fe")
+ oil(ne):0 col(ne):0 gas(ne):0
      O:PY(i)                      Q:vom(i)
      I:PY(j)$ (not fe(j))         Q:vfi(j,i)          e:$ele(j)
      I:PL                          Q:vfm("lab",i)      va:
      I:PK                          Q:vfm("cap",i)      va:
      I:PY(fe)                     Q:vfi(fe,i)          fe.tl:
      I:PCO2#(fe)$clim             Q:fico2(fe,i)      P:1e-5      fe.tl:
```

An energy model with emissions (ENERGY2.gms)

- Emissions allowances are endowed to the house1 → revenue from the cap-and-trade program is returned to households

```
*      Household demand
$DEMAND:HH
      D:PW          Q:vthp
      E:PL          Q:efh("lab")
      E:PK          Q:efh("cap")
      E:PR(i)       Q:vfm("res",i)
      E:PCO2$clim   Q:clim
```

The quantity of emissions rights is controlled by `clim`

An energy model with emissions (ENERGY2.gms)

- The default level for all GAMS variables is 1. To replicate the benchmark dataset, we need to set the price of CO2 emission equal to zero.

```
PCO2.L = 0;
```

- ENERGY2 implements a 20% reduction in CO2 emissions

```
clim = 0.8*sum(i, fco2(i));  
$include ENERGY2.gen  
solve ENERGY2 using mcp;
```

An energy model with emissions (ENERGY2.gms)

- Results reporting includes the quantity of emissions and the CO₂ price

PARAMETER

EMISSIONS(*) CO2 emissions - million metric tons

PRICECO2(*) CO2 price \$ per metric ton;

EMISSIONS("BASE") = sum(i, fco2(i));

EMISSIONS("SIM") = sum((fe,i)\$vfi(fe,i),
fico2(fe,i)*FFE.L(fe,i)/vfi(fe,i))
+ sum(fe, hhco2(fe)*HFE.L(fe)/vhp(fe));

Benchmark emissions of from
fuel f_e used in sector i

EMISSIONS("%C") = 100*(EMISSIONS("SIM") - EMISSIONS("BASE")) / EMISSIONS("BASE");

PRICECO2("SIM") = 1e3*PCO2.L/PW.L;

Ratio of fuel input f_e used in sector i in
the simulation to that in the benchmark

DISPLAY OUTPUT, WELFARE, EMISSIONS, PRICECO2;

EXECUTE_UNLOAD "ENERGY2_OUTPUT.gdx"

An energy model with emissions (ENERGY2.gms)

- Results reporting includes the quantity of emissions and the CO₂ price

PARAMETER

EMISSIONS(*) CO2 emissions - million metric tons

PRICECO2(*) CO2 price \$ per metric ton;

EMISSIONS("BASE") = sum(i, fco2(i));

EMISSIONS("SIM") = sum((fe,i)\$vfi(fe,i),
fico2(fe,i)*FFE.L(fe,i)/vfi(fe,i))
+ sum(fe, hhco2(fe)*HFE.L(fe)/vhp(fe));

Economic data are in billions and
emissions data are in millions → multiply
by (1e9/1e6)

EMISSIONS("%C") = 100*(EMISSIONS("SIM")-EMISSIONS("BASE"))/EMISSIONS("BASE");

PRICECO2("SIM") = 1e3*PCO2.L/PW.L;

Divide by a consumer price index so
that the *numeraire* choice does not
influence the emissions price

DISPLAY OUTPUT, WELFARE, EMISSIONS, PRICECO2;

EXECUTE_UNLOAD "ENERGY2_OUTPUT.gdx"

All model output is sent to a gdx file

Exercises

1. Implement a 30% reduction in emissions and analyze the results.
2. When there a 20% reduction in emissions, change the value of $\sigma("e_{kl}')$ from 0.5 to 1 and analyze changes in the CO₂ price and welfare.